Online Appendix for "A welfare analysis of tax structures with love-of-variety preferences"

A Specific Taxation and Ad Valorem and Results

Proof. ***Marginal Excess Burden Formula for specific tax*** $\frac{dW}{dt}$**.**

Let the total welfare to be the sum of consumer surplus, profits and government tax revenues.

$$W(p(t),t,J(t)) = \underbrace{u(Q_L(t),J(t)) - (p(t)+t)Q}_{CS} + \underbrace{p(t)Q_L(t) - Jc\left(\frac{Q_L(t)}{J(t)}\right) - J(t)F}_{J\pi} + \underbrace{tQ_L(t)}_{R}$$

By totally differentiating $W_L(t) = W(p(t),t,J(t))$ we obtain

$$\begin{aligned} \frac{dW_L}{dt} &= \left(\frac{\partial u}{\partial Q}(Q_0,J_0) - c'(q_0)\right)\frac{dQ_L}{dt} + \left(\frac{\partial u}{\partial J}(Q_0,J_0) - c(q_0) - F + q_0c'(q_0)\right)\frac{dJ}{dt} \\ &= (p_0 + \theta t_0 - c'(q_0))\frac{dQ_L}{dt} + (\Lambda_0 + \pi_0 - [p_0 - c'(q_0)] * q_0)\frac{dJ}{dt} \end{aligned} \tag{1}$$

where we used the first-order approximation from Chetty, Looney and Kroft (2009) $\frac{\partial u}{\partial J}(Q_0,J_0) = p_0 + \theta_t t_0$, we used our definition of variety effect $\Lambda_0 = \frac{\partial u}{\partial J}(Q_0,J_0)$ and profits $\pi_0 = p_0q_0 - c(q_0) - F$. When $t_0 = 0$, $p_0 = c'(q^*)$ and $\Lambda_0 = -\pi_0$, we get $\frac{dW_L}{dt} = 0$ which is the first-best outcome. □

Proof. ***Lemma 1.***

Let $\pi = pq - c(q) = 0$ be the free-entry condition of firms. Then $\frac{d\pi}{dt} = 0$ implies that$(p - mc)\frac{dq}{dt} = -q\frac{dp}{dt}$ and so $\frac{p-mc}{p} = -\frac{q/t}{p/t}\frac{\frac{dp}{dt}}{\frac{dq}{dt}}$. □

Proof. ***Proposition 1 in general case without Assumption 3.***

Let $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\epsilon_D J\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J\partial Q}q\nu_q\right)}{(p(1+\tau)+t)}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right)$.[1] The firm stability

[1]This becomes $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right)$ under Assumption 3 of parallel demands.

conditions $\frac{\partial^2 \pi_j}{\partial p_j^2} < 0$ and $\frac{\partial \pi_j}{\partial J} < 0$, are respectively equivalent to $1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} > 0$ and $\Delta > 0$, where $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$. Here, Δ and ϵ_D^* are written in the general form that depends on both the specific tax rate t and the ad valorem tax rate τ for convenience, however from Proposition 1 we set $\tau = 0$.

By Lemma 1, we have $\frac{dPS}{dt} = 0$. Therefore substituting this into equation (1) we obtain:

$$\frac{dW}{dt} = \Lambda_0 \frac{dJ}{dt} - Q_0 \frac{dp}{dt} + \theta_t t_0 \frac{dQ_L}{dt}$$

From the behavioral equation of consumers $wtp(Q) = p + \theta_t t$, we have

$$mwtp(Q, J)\frac{dQ}{dt} + \frac{\Lambda}{Q}\frac{dJ}{dt} = \frac{dp}{dt} + \theta_t \tag{2}$$

In addition, from the free-entry condition, $(p - mc)\frac{dq}{dt} = -q\frac{dp}{dt}$, and firm's first-order condition, $p - mc = ms(Q)\frac{\nu_q}{J}$, we have

$$mwtp(Q, J)\nu_q \frac{dq}{dt} = \frac{dp}{dt} \tag{3}$$

Combining this with the behavioral equation above, and letting $mwtp(Q, J) = mwtp(Q)$ for simplicity, we have

$$\begin{aligned} mwtp(Q)\nu_q \frac{dq}{dt} &= mwtp(Q)\frac{dQ}{dt} + \frac{\Lambda}{Q}\frac{dJ}{dt} - \theta_t \\ &= mwtp(Q)\left(J\frac{dq}{dt} + q\frac{dJ}{dt}\right) + \frac{\Lambda}{Q}\frac{dJ}{dt} - \theta_t \end{aligned} \tag{4}$$

where the second line follows from substituting $\frac{dQ}{dt} = J\frac{dq}{dt} + q\frac{dJ}{dt}$. Therefore,

$$\frac{dq}{dt} = \frac{\theta_t - \left(\frac{\Lambda}{Q} + q * mwtp(Q)\right)\frac{dJ}{dt}}{mwtp(Q)(J - \nu_q)} \tag{5}$$

Using now $\frac{dq}{dt} = \frac{\partial q}{\partial t} + \frac{\partial q}{\partial J}\frac{dJ}{dt}$ (note that $\frac{\partial q}{\partial t} = \frac{dq}{dt}\Big|_J$) we can get

$$\frac{dJ}{dt} = \frac{\theta_t - (J-\nu_q)mwtp(Q)\frac{\partial q}{\partial t}}{\frac{\Lambda}{Q} + q * mwtp(Q) + (J-\nu_q)mwtp(Q)\frac{\partial q}{\partial J}} \tag{6}$$

From Kroft et al. (2020), we have

$$\frac{\partial q}{\partial t} = \frac{dq}{dt}\bigg|_J = \frac{1}{Jmwtp(Q)}\left(\rho_t^{SR} + \theta_t - 1\right) = \frac{\omega_t^{SR}\theta_t}{Jmwtp(Q)} \tag{7}$$

where $\rho_t^{SR} = 1 - (1-\omega_{SR})\,\theta_t$ and $\omega_{SR} = \frac{1}{1+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}$, where $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$ (short-run passthrough is taken from Kroft et al. (2020), for section 3 $\tau = 0$, while for section 4 $t = 0$, however ω_{SR} and ϵ_D^* can be written in this general form for convenience).

Finally, fix t, and differentiate the first-order confition with respect to J to get:

$$\frac{\Lambda}{Q} + mwtp(Q)\left(q + J\frac{\partial q}{\partial J}\right) - c''(q)\frac{\partial q}{\partial J} = -\frac{\partial q}{\partial J}mwtp(Q)\nu_J - q\nu_J mwtp'(Q)\left(q + J\frac{\partial q}{\partial J}\right) - \frac{\partial^2 P}{\partial J \partial Q}q\nu_q$$

where we have assumed that $\frac{\partial \nu}{\partial J} = 0$. Further simplifying yields:

$$\frac{\partial q}{\partial J} = -\frac{\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q}q\nu_q + mwtp(Q)q + q^2\nu_q mwtp'(Q)}{(J+\nu_q)mwtp(Q) - c''(q) + Jq\nu_q mwtp'(Q)} \tag{8}$$

Rearranging equation (8), the denominator is equal to $J * mwtp(Q) * \left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)$, and so we get:

$$\frac{\partial q}{\partial J} = -\frac{\omega_{SR}}{J * mwtp(Q)}\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q}q\nu_q\right) - \frac{q}{J}\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) \tag{9}$$

Note:

$$\begin{aligned}\omega_{SR}\frac{\nu_q}{J}\Delta = &\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q}q\nu_q\right)\left(1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)\right) \\ &+ q * mwtp(Q)\left(1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)\end{aligned}$$

Substituting equation (9) and equation (7) into equation (6), we get:

$$\frac{dJ}{dt} = \theta_t \left(\frac{1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)}{\omega_{SR}\frac{\nu_q}{J}\Delta} \right)$$

, and substituting $\frac{dJ}{dt}$ into equation (5), we obtain:

$$\frac{dq}{dt} = \frac{\theta_t q}{J} \left(\frac{\frac{\nu_q}{J} - \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}{\frac{\nu_q}{J}\Delta} \right)$$

Finally, from equation (3) and the expression for $\frac{dq}{dt}$ we have:

$$\begin{aligned} \rho_t &= 1 + mwtp(Q,J)\nu_q \frac{dq}{dt} \\ &= \frac{\frac{\nu_q}{J}\left[2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + (1-\theta_t)\left(\frac{\nu_q}{J} - \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right] - \frac{\epsilon_D J}{p+t}\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q} q\nu_q\right)\left[\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]}{\frac{\nu_q}{J}\left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\epsilon_D J}{p+t}\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q} q\nu_q\right)\left[\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]} \end{aligned}$$

When we impose Assumption 3 (parallel demands). we obtain that $\frac{\partial P}{\partial J}(Q,J) = \frac{\Lambda}{Q}$ and $\frac{\partial^2 P}{\partial J \partial Q} = 0$. Therefore, equation (9) is translated to

$$\frac{\partial q}{\partial J} = -\frac{\omega_{SR}\Lambda}{JQ * mwtp(Q)} - \frac{q}{J}\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) \tag{10}$$

And following the same steps we obtain:

$$\frac{dq}{dt} = -\frac{\theta_t q \epsilon_D}{p+t}\left(\frac{\frac{\nu_q}{J} - \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}{\frac{\nu_q}{J}\left(2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - \frac{\Lambda \epsilon_D}{(p+t)q}\left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)} \right) \tag{11}$$

$$\frac{dJ}{dt} = -\frac{\theta_t J \epsilon_D}{p+t}\left(\frac{\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}{\frac{\nu_q}{J}\left(2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - \frac{\Lambda \epsilon_D}{(p+t)q}\left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)} \right) \tag{12}$$

$$\rho_t = \frac{\frac{\nu_q}{J}\left[2+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}-(1-\theta_t)\left(\frac{\nu_q}{J}-\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right]-\frac{\Lambda\epsilon_D}{(p+t)q}\left[\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]}{\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p+t)q}\left[\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]} \tag{13}$$

□

Proof. ***Corollary 1.***

The proof is immediate by setting $\theta_t = 1$, $\Lambda_0 = 0$ and $t_0 = 0$ into the conditions of Proposition 1. □

Proof. ***Corollary 2.***

First, the overshifting condition is given by:

$$\begin{aligned}
&\frac{dp}{dt} \geq 0 \\
&\Leftrightarrow mwtp(Q,J)\nu_q \frac{dq}{dt} \leq 0 \\
&\Leftrightarrow \frac{\frac{\nu_q}{J}-\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}{\frac{\nu_q}{J}\left(2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)-\frac{\Lambda\epsilon_D}{(p+t)q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)} \geq 0 \\
&\Leftrightarrow 1-\frac{1}{\epsilon_{ms}} \geq 0
\end{aligned}$$

where we have used that $\Delta \geq 0$ by stability.

Next, note that the second-order conditions imply

$$\frac{\Lambda}{Q}\left(1-\omega_{SR}\left(1-\frac{\nu_q}{J}\right)\right)+q*mwtp(Q)\left(1-\omega_{SR}\left(1-\frac{\nu_q}{J}\right)\left(1-\frac{\nu_q}{J}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)<0$$

and $1+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}>0$. Thus,

$$sign\left(\frac{dJ}{dt}\right) = -sign\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)$$

and

$$sign\left(\frac{dq}{dt}\right) = sign\left(\frac{1}{\epsilon_{ms}}-1\right)$$

It follows that

$$\begin{aligned}\frac{dW}{dt} \geq 0 \Leftrightarrow \frac{dq}{dt} \geq \frac{\Lambda\epsilon_D}{\nu_q p}\frac{dJ}{dt} \\ \Leftrightarrow 1 - \frac{1}{\epsilon_{ms}} \leq \frac{\Lambda\epsilon_D}{qp\left(\frac{\nu_q}{J}\right)^2}\left[\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]\end{aligned}$$

□

Proof. ***Lemma 2.***

From the behavioral equation $wtp(Q) = P(Q, J) = p + \theta t$, we can express price as a function of J and t. Then we have

$$p(J, t) = P(Q(J, t), J) - \theta t$$

Therefore,

$$\begin{aligned}\frac{\partial p}{\partial J} &= \frac{\partial P}{\partial J} + mwtp(Q, J)\frac{\partial Q}{\partial J} \\ &= \frac{\Lambda}{Q} + q * mwtp(Q, J) + mwtp(Q, J) * J * \frac{\partial q}{\partial J} \\ &= \left[\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right]\end{aligned}$$

From the proof of Proposition 1, we also have that:

$$\begin{aligned}\frac{\partial q}{\partial J} &= -\frac{\frac{\Lambda}{Q} + mwtp(Q)q + q^2\nu_q mwtp'(Q)}{(J + \nu_q)mwtp(Q) - c''(q) + Jq\nu_q mwtp'(Q)} \\ &= -\frac{\omega_{SR}\Lambda}{JQ * mwtp(Q)} - \frac{q}{J}\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\end{aligned}$$

Therefore,

$$\frac{\partial p}{\partial J} = \left[\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right]$$
$$\frac{J}{q}\frac{\partial q}{\partial J} = -\omega_{SR}\left[1 - \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) - \frac{\Lambda\epsilon_D}{(p+t)q}\right]$$

□

Proof. ***Corollary 3.***

Observe that

$$\begin{aligned}
\frac{\partial \pi}{\partial t} &= \frac{\partial p}{\partial t}q + (p - mc)\frac{\partial q}{\partial t} \\
&= \frac{\partial p}{\partial t}q + \frac{\nu_q}{J\epsilon_D^*}\frac{\partial q}{\partial t}p \\
&= \left(\rho_t^{SR} - 1\right)q + \frac{\nu_q}{J\epsilon_D^*}p\frac{\rho_t^{SR} - 1 + \theta_t}{Jmwtp(Q)} \\
&= q\theta_t\omega_{SR}\left(1 - \frac{\nu_q}{J} - \frac{1}{\omega_t^{SR}}\right) \\
&= -q\theta_t\omega_{SR}\left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)
\end{aligned}$$

where the term in parenthesis is the numerator in equation (12). This implies that given the denominator is positive by stability, then:

$$sign\left(\frac{\partial \pi}{\partial t}\right) = sign\left(\frac{dJ}{dt}\right)$$

From the behavioral equation $wtp(Q) = P(Q, J) = p + \theta t$, we can express price as a function of J and t. Then we have

$$p(J, t) = P(Q(J, t), J) - \theta t$$

Therefore, using Lemma 2 for the second line we obtain:

$$\begin{aligned}\frac{dp}{dt} &= \frac{\partial p}{\partial J}\frac{dJ}{dt} + \frac{\partial p}{\partial t} \\ &= \left(\frac{\Lambda}{Q} + mwtp(Q,J)\frac{\partial Q}{\partial J}\right)\frac{dJ}{dt} + (\rho_{SR} - 1)\end{aligned}$$

which implies that

$$\rho_t - \rho_t^{SR} = \frac{\partial p}{\partial J}\frac{dJ}{dt}$$

Then we can express the difference between long-run and short-trun pass-through as:

$$\begin{aligned}\rho_t - \rho_t^{SR} &= \left(\frac{\Lambda}{Q} - \frac{\omega_{SR}\Lambda}{Q} + q * mwtp(Q,J) - q * mwtp(Q,J) * \omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)\frac{dJ}{dt} \\ &= \left((1-\omega_{SR})\frac{\Lambda}{Q} + \frac{p+t}{J\epsilon_D} * \left(\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - 1\right)\right)\frac{dJ}{dt} \\ &= \left((1-\omega_{SR})\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \omega_{SR} * \left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}\right)\right)\frac{dJ}{dt} \\ &= \left((1-\omega_{SR})\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \omega_{SR} * \left(1 - \omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)\right)\frac{dJ}{dt} \\ &= \frac{1}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}\left(\left(\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}\right)\right)\frac{dJ}{dt}\end{aligned}$$

Therefore,

$$sign\left(\rho_t - \rho_t^{SR}\right) = -sign\left(\left(\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}\right)\right) * sign\left(\frac{dJ}{dt}\right)$$

Finally, under the conditions of Corollary 3, we can sign part of the following expression as follows:

$$\rho_t - \rho_t^{SR} = \frac{1}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}} \left(\left(\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) \frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} \right) \right) \frac{dJ}{dt}$$
$$= \frac{1}{1 + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}} \underbrace{\left(\left(\frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) \frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \left(\frac{\nu_q}{J} \right) \right)}_{<0} \frac{dJ}{dt}$$

which implies

$$sign\left(\rho_t - \rho_t^{SR}\right) = -sign\left(\frac{\partial \pi}{\partial t}\right)$$

□

Proof. ***Lemma 3.***

Let $\pi = pq - c(q) = 0$ by the free-entry condition. Then $\frac{d\pi}{d\tau} = 0$ implies $(p - mc)\frac{dq}{d\tau} = -q\frac{dp}{d\tau}$ and so $\frac{p-mc}{p} = -\frac{q/\tau}{p/\tau}\frac{\frac{dp}{d\tau}}{\frac{dq}{d\tau}}$. □

Proof. ***Proposition 2 with Assumption 3.***

We will provide a proof of Proposition 2 under parallel demands and then discuss at the end how the formulas change without parallel demands. Note that for marginal excess burden, we do not require Assumption 3.

Consider a change in the tax from τ_0 to τ_1. A first-order approximation to the marginal excess burden of taxation is:

$$\frac{dW}{d\tau} = \underbrace{(p_0(1 + \theta_\tau \tau_0) - c'(q_0))\frac{dQ_L}{d\tau}}_{\text{Quantity effect}} + \underbrace{(\Lambda_0 + \pi_0 - [p_0 - c'(q_0)] * q_0)\frac{dJ}{d\tau}}_{\text{Diversity effect}} \tag{14}$$

Under Lemma 3, the marginal excess burden of taxation is given by:

$$\frac{dW}{d\tau} = \Lambda_0 \frac{dJ}{d\tau} - Q_0 \frac{dp}{d\tau} + \theta_\tau \tau_0 p_0 \frac{dQ_L}{d\tau} \tag{15}$$

Willingness-to-pay with ad valorem taxes takes the form $wtp(Q) = p(1+\theta_\tau \tau)$, so $mwtp(Q)\frac{dQ}{d\tau} + \frac{\partial P}{\partial J}\frac{dJ}{d\tau} = \frac{dp}{d\tau}(1+\theta_\tau \tau) + p\theta_\tau$. With the parallel demands assumption, we have $\frac{\partial P}{\partial J} = \frac{\Lambda}{Q}$. We also have the free entry-condition $(p - mc)\frac{dq}{d\tau} = -q\frac{dp}{d\tau}$, and the firm's first-order condition $p - mc = -\frac{\nu_q}{J(1+\theta_\tau \tau)} mwtp(Q)Q$. Therefore, we have:

$$\nu_q * mwtp(Q)\frac{dq}{d\tau} = (1+\theta_\tau \tau)\frac{dp}{d\tau} \tag{16}$$

which implies:

$$\frac{dq}{d\tau} = \frac{p\theta_\tau - \left(\frac{\Lambda}{Q} + q * mwtp(Q)\right)\frac{dJ}{d\tau}}{mwtp(Q)\left(1 - \frac{\nu_q}{J}\right)} \tag{17}$$

Using now $\frac{dq}{d\tau} = \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial J}\frac{dJ}{d\tau}$ (Here $\frac{\partial q}{\partial \tau} = \left.\frac{dq}{d\tau}\right|_J$). we get

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau + \left(\nu_q - J\right) mwtp(Q)\frac{\partial q}{\partial \tau}}{\frac{\Lambda}{Q} + q * mwtp(Q) + \left(J - \nu_q\right)\frac{\partial q}{\partial J}} \tag{18}$$

We also have

$$\frac{\partial q}{\partial \tau} = \left.\frac{dq}{d\tau}\right|_J = \frac{1}{Jmwtp(Q)}\left(\theta_\tau mc * \omega_{SR}\right)$$

where $\rho_\tau^{SR} = 1 - \left(1 - \omega_{SR}\frac{mc}{p}\right)\theta_\tau$ and $\omega_{SR} = \frac{1}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}$. Moreover,

$$\frac{\partial q}{\partial J} = -\frac{\Lambda}{Q}\frac{\omega_{SR}}{J * mwtp(Q)} - \frac{q\omega_{SR}}{J}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) \tag{19}$$

Therefore, substituting $\frac{\partial q}{\partial \tau}$ and $\frac{\partial q}{\partial J}$ into equation (18)we have

$$\begin{aligned}\frac{dJ}{d\tau} &= \theta_\tau \left(\frac{p - mc * \omega_{SR}\left(1-\frac{\nu_q}{J}\right)}{\frac{\Lambda}{Q}\left(1-\omega_{SR}\left(1-\frac{\nu_q}{J}\right)\right) + q * mwtp(Q)\left(1-\omega_{SR}\left(1-\frac{\nu_q}{J}\right)\left(1-\frac{\nu_q}{J}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)} \right) \\
&= p\theta_\tau \left(\frac{\left(1+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - \left(1-\frac{\frac{\nu_q}{J}}{\epsilon_D^*}\right)\left(1-\frac{\nu_q}{J}\right)}{\frac{\Lambda}{Q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) + q * mwtp(Q)\left(\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]\right)} \right) \\
&= -\frac{\theta_\tau J \epsilon_D}{1+\tau} \left(\frac{\frac{\nu_q}{J}\left(1+\frac{1}{\epsilon_D^*}-\frac{\frac{\nu_q}{J}}{\epsilon_D^*}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}+\frac{1}{\epsilon_{ms}}\right)}{\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda\epsilon_D}{(1+\tau)pq}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)} \right) \qquad (20)\end{aligned}$$

Recall $\Delta = \left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(1+\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}+\frac{1}{\epsilon_{ms}}\right)$. Substituting equation (20) into equation (17), then:

$$\begin{aligned}\frac{dq}{d\tau} &= \frac{-\theta_\tau \omega_{SR}}{Jmwtp(Q)} \left(\frac{\frac{\Lambda}{Q}(p-mc) + q * mwtp(Q)\left(p\left(1-\frac{\nu_q}{J}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)-mc\right)}{\omega_{SR}\frac{\nu_q}{J}\Delta} \right) \\
&= \frac{-p\theta_\tau}{Jmwtp(Q)} \left(\frac{\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} - \frac{\Lambda\epsilon_D}{(1+\tau)pq}\frac{\frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{\nu_q}{J}\Delta} \right)\end{aligned}$$

Finally,

$$\begin{aligned}\rho_\tau &= \frac{1}{p}\frac{1+\tau}{1+\theta_\tau \tau}\nu_q mwtp(Q)\frac{dq}{d\tau} + 1 \\
&= -\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)} \left(\frac{\frac{\Lambda}{Q}\left(\frac{p-mc}{p}\right) + q * mwtp(Q)\left(\frac{p-mc}{p}-\frac{\nu_q}{J}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)}{\frac{\nu_q}{J}\Delta} \right) + 1 \\
&= \frac{\frac{\nu_q}{J}\Delta - \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left(\frac{p-mc}{p}-\frac{\nu_q}{J}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) + \frac{\Lambda\epsilon_D}{(1+\tau)pq}\left(\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{p-mc}{p}\right)}{\frac{\nu_q}{J}\Delta}\end{aligned}$$

Using $\frac{p-mc}{p} = \frac{\frac{\nu_q}{J}}{\epsilon_D^*}$, we obtain:

$$\rho_\tau = \frac{\Delta - \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right) + \frac{\Lambda\epsilon_D}{(1+\tau)pq}\left(\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{1}{\epsilon_D^*}\right)}{\Delta}$$

In the case where Assumption 3 does not hold, analogous results can be derived by substituting $\frac{\Lambda}{q}$ with $J(\frac{\partial P}{\partial J}+\frac{\partial^2 P}{\partial J \partial Q} q\nu_q)$. The proof is completely analogous to that of Proposition 1 without Assumption 3. □

Proof. ***Corollary 4.***

This follows immediately by setting $\theta = 1$, $\Lambda_0 = 0$ and $\tau_0 = 0$ into the conditions of Proposition 2. □

Proof. ***Corollary 5.***

Assume that $\nu_q \in (0, J]$, $\theta_\tau \in [0,1]$, and that $\pi_0 = 0$. We derive each of the results stated in the Corollary:

1. Overshifting: a small tax increases producer prices if and only if:

$$\frac{dp}{d\tau} \geq 0$$
$$\Leftrightarrow \rho_\tau \geq 1$$
$$\Leftrightarrow -\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) + \frac{\Lambda\epsilon_D}{(1+\theta_\tau\tau)pq}\left(\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{\frac{\nu_q}{J}}{\epsilon_D}\right) \geq 0$$
$$\Leftrightarrow -\frac{1}{\epsilon_D^*} + 1 - \frac{1}{\epsilon_{ms}} \geq -\frac{\Lambda\epsilon_D}{(1+\theta_\tau\tau)pq}\frac{1}{\epsilon_D}$$
$$\Leftrightarrow 1 - \frac{1}{\epsilon_{ms}} \geq \frac{1}{\epsilon_D^*} - \frac{\Lambda_0}{p_0q_0(1+\theta_\tau\tau_0)}$$

2. Starting from no tax $\tau_0 = 0$, introducing a small specific tax, increases welfare if and only if:

$$\frac{dW}{d\tau} = \Lambda_0\frac{dJ}{d\tau} - Q_0\frac{dp}{d\tau} \geq 0 \Leftrightarrow \frac{\Lambda_0}{p_0Q_0}\frac{dJ}{d\tau} \geq \rho_\tau - 1$$
$$\Leftrightarrow \frac{1}{\epsilon_D^*} + \frac{1}{\epsilon_{ms}} - 1 \geq \frac{\Lambda_0\epsilon_D}{p_0q_0\frac{\nu_q}{J_0}}\left[1 + \frac{1}{\epsilon_D^*} + \frac{1}{\epsilon_{ms}} + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\epsilon_S\frac{\nu_q}{J_0}}\right]$$

3. Therefore, if $\Lambda_0 = 0$, starting from no tax $\tau_0 = 0$, introducing a small tax, increases

welfare if and only if there is no overshifting:

$$\frac{dW}{d\tau} \geq 0 \Leftrightarrow \frac{dp}{d\tau} \leq 0 \Leftrightarrow \frac{1}{\epsilon_D^*} + \frac{1}{\epsilon_{ms}} - 1 \geq 0$$

□

Proof. ***Lemma 4.***

The proof is analogous to Lemma 2. The only modification is that the behavioral equation for ad valorem taxation $p(J,t) = \frac{P(Q(J,t),J)}{1+\theta_\tau \tau}$ implies a rescaling is needed for $\frac{\partial p}{\partial J}$. □

Proof. ***Corollary 6.***

Note that:

$$\begin{aligned}
\frac{\partial \pi}{\partial \tau} &= \frac{\partial p}{\partial \tau} q + (p - mc)\frac{\partial q}{\partial \tau} \\
&= \frac{\partial p}{\partial \tau} q - \frac{\nu_q}{J}\frac{Qmwtp(Q)}{1+\theta_\tau \tau}\frac{\partial q}{\partial \tau} p \\
&= \left(\rho_\tau^{SR} - 1\right) pq - \frac{\nu_q}{J}\frac{Qmwtp(Q)}{1+\theta_\tau \tau}\frac{\frac{\partial p}{\partial t}(1+\theta_\tau \tau) + p\theta_\tau}{Jmwtp(Q)} \\
&= \left(\rho_\tau^{SR} - 1\right) pq - \frac{\nu_q}{J}\frac{pq}{1+\theta_\tau \tau}\left(\left(\rho_\tau^{SR} - 1\right)(1+\theta_\tau \tau) + \theta_\tau\right) \\
&= pq\left[\left(\rho_\tau^{SR} - 1\right)\left(1 - \frac{\nu_q}{J}\right) - \frac{\nu_q}{J}\left(\frac{\theta_\tau}{1+\theta_\tau \tau}\right)\right] \\
&= pq\theta_\tau\left[\left(\frac{mc}{p}\omega_{SR} - 1\right)\left(1 - \frac{\nu_q}{J}\right) - \frac{\nu_q}{J}\left(\frac{1}{1+\theta_\tau \tau}\right)\right] \\
&= -pq\theta_\tau\omega_{SR}\left[\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - \left(1 - \frac{\frac{\nu_q}{J}}{\epsilon_D^*}\right)\left(1 - \frac{\nu_q}{J}\right) + \frac{\nu_q}{J}\left(\frac{\theta_\tau \tau}{1+\theta_\tau \tau}\right)\right] \\
&= -pq\theta_\tau\omega_{SR}\left(\frac{\nu_q}{J}\left(1 + \frac{1-\frac{\nu_q}{J}}{\epsilon_D^*} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) + \frac{\nu_q}{J}\left(\frac{\theta_\tau \tau}{1+\theta_\tau \tau}\right)\right)
\end{aligned}$$

and

$$\frac{dJ}{d\tau} = -\frac{\theta_\tau J\epsilon_D}{1+\tau}\left(\frac{\frac{\nu_q}{J}\left(1 + \frac{1}{\epsilon_D^*} - \frac{\frac{\nu_q}{J}}{\epsilon_D^*} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right)}{\frac{\nu_q}{J}\left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda\epsilon_D}{(1+\tau)pq}\left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)}\right)$$

which implies

$$sign\left(\frac{\partial \pi}{\partial \tau}\right) = sign\left(\frac{dJ}{d\tau}\right)$$

Finally, we also have

$$\begin{aligned}
\frac{dp}{d\tau} &= \frac{\partial p}{\partial J}\frac{dJ}{dt} + \frac{\partial p}{\partial t} \\
&= \frac{1}{1+\theta_\tau \tau}\left(\frac{\partial P}{\partial J} + mwtp(Q,J)\frac{\partial Q}{\partial J}\right)\frac{dJ}{d\tau} + \frac{1}{1+\theta_\tau \tau}\left(mwtp(Q,J)\frac{\partial Q}{\partial \tau} - \theta_\tau p\right) \\
&= \frac{1}{1+\theta_\tau \tau}\left(\frac{\Lambda}{Q} + mwtp(Q,J)\frac{\partial Q}{\partial J}\right)\frac{dJ}{d\tau} + \left(\frac{\partial p}{\partial \tau}\right)
\end{aligned}$$

which implies

$$\rho_{LR}^{\tau} - \rho_{SR}^{\tau} = \frac{1+\tau}{1+\theta_\tau \tau}\left(\frac{\Lambda}{Q} + q * mwtp(Q,J) + mwtp(Q,J) * J * \frac{\partial q}{\partial J}\right)\frac{1}{p}\frac{dJ}{d\tau}$$

and so

$$\begin{aligned}
\rho_\tau - \rho_\tau^{SR} &= \frac{\frac{1+\tau}{1+\theta_\tau\tau}}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}\left(\left(\frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\frac{\Lambda}{Q} - \frac{p(1+\tau)}{J\epsilon_D} * \left(\frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S}\right)\right)\frac{dJ}{d\tau} \\
&= \frac{\frac{1+\tau}{1+\theta_\tau\tau}}{1 + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}\underbrace{\left(\left(\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D} * \left(\frac{\nu_q}{J}\right)\right)}_{<0}\frac{dJ}{d\tau}
\end{aligned}$$

which implies that:

$$sign\left(\rho_\tau - \rho_\tau^{SR}\right) = -sign\left(\frac{\partial \pi}{\partial \tau}\right)$$

□

B Comparison between Ad Valorem and Specific Taxation

We begin by considering the reduced-form effects of taxes in order to compare ad valorem to specific taxation. Throughout we will make use of the definitions $\epsilon_D = -\frac{p(1+\tau)+t}{Qmwtp(Q)}$. $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$, and $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) > 0$ for the stability condition:

$$\rho_t = \frac{\Delta + \theta_t \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right)}{\Delta}$$

$$\rho_\tau = \frac{\Delta + \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left(1 - \frac{1}{\epsilon_{ms}} + \frac{1}{\epsilon_D^*}\left(\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q} - 1\right)\right)}{\Delta}$$

$$\frac{dq}{dt} = -\frac{\theta_t q\epsilon_D}{p(1+\tau)+t}\left(\frac{1 - \frac{1}{\epsilon_{ms}}}{\Delta}\right)$$

$$\frac{dq}{d\tau} = -\frac{\theta_\tau pq\epsilon_D}{p(1+\tau)+t}\left(\frac{1 - \frac{1}{\epsilon_{ms}} - \frac{1}{\epsilon_D^*} + \frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\frac{1}{\epsilon_D^*}}{\Delta}\right)$$

$$\frac{dJ}{dt} = -\frac{\theta_t J\epsilon_D}{p(1+\tau)+t}\left(\frac{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}}{\Delta}\right)$$

$$\frac{dJ}{d\tau} = -\frac{\theta_\tau pJ\epsilon_D}{p(1+\tau)+t}\left(\frac{1 + \frac{1}{\epsilon_D^*} - \frac{\frac{\nu_q}{J}}{\epsilon_D^*} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}}{\Delta}\right)$$

$$\frac{dQ}{dt} = -\frac{\theta_t Q\epsilon_D}{p(1+\tau)+t}\left(\frac{2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}}{\Delta}\right)$$

$$\frac{dQ}{d\tau} = -\frac{\theta_\tau pQ\epsilon_D}{p(1+\tau)+t}\left(\frac{2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \left(\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q} - \frac{\nu_q}{J}\right)\frac{1}{\epsilon_D^*}}{\Delta}\right)$$

$$\frac{dW}{dt} = \Lambda\frac{dJ}{dt} + \theta_t t\frac{dQ}{dt} - Q\frac{dp}{dt}$$

$$\frac{dW}{d\tau} = \Lambda\frac{dJ}{d\tau} + \theta_\tau\tau p\frac{dQ}{d\tau} - Q\frac{dp}{d\tau}$$

$$\frac{dR}{dt} = Q + t\frac{dQ}{dt}$$

$$\frac{dR}{d\tau} = pQ + \tau p\frac{dQ}{d\tau} + \tau Q\frac{dp}{d\tau}$$

Proof. ***Proposition 3.*** Rewrite ρ_τ as:

$$\rho_\tau = \frac{\frac{\nu_q}{J}\left[2+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}-\left(1-\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\right)\left(\frac{\nu_q}{J}-\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right]}{\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)}$$

$$-\frac{\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)+\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{\left(\frac{\nu_q}{J}\right)^2}{\epsilon_D^*}\left[\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}-1\right]}{\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)}$$

Then, observe that for $\theta_t = \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}$ (for example if $\theta_t = \theta_\tau$ and $\tau = 0$) then

$$\rho_\tau - \rho_t = \frac{\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{\left(\frac{\nu_q}{J}\right)^2}{\epsilon_D^*}\left[\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}-1\right]}{\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left[\frac{\nu_q}{J}+\frac{\epsilon_D-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]}$$

so

$$\rho_\tau > \rho_t \Leftrightarrow \frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q} > 1 \Leftrightarrow \frac{\Lambda}{Q} + q * mwtp(Q) > 0$$

We now consider the marginal cost of public funds (MCPF) starting from zero initial taxes.

$$R = \tau p Q + tQ$$

$$\begin{aligned} MCPF_t &= -\frac{\Lambda\frac{dJ}{dt}+\theta_t t\frac{dQ}{dt}-Q\frac{dp}{dt}}{Q+t\frac{dQ}{dt}} \\ &= -\frac{\Lambda}{Q}\frac{dJ}{dt}+\frac{dp}{dt} \\ &= -\frac{\Lambda}{Q}\frac{dJ}{dt}+\rho_t-1 \end{aligned}$$

$$\begin{aligned} MCPF_\tau &= -\frac{\Lambda\frac{dJ}{d\tau}+\theta_\tau\tau p\frac{dQ}{d\tau}-Q\frac{dp}{d\tau}}{pQ+\tau p\frac{dQ}{d\tau}+\tau Q\frac{dp}{d\tau}} \\ &= -\frac{\Lambda}{pQ}\frac{dJ}{d\tau}+\rho_\tau-1 \end{aligned}$$

Furthermore,

$$\frac{dJ}{dt} = \frac{\theta_t}{\frac{\Lambda}{Q} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} \frac{dp}{dt}$$

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau}{\frac{\Lambda}{Q} + q * mwtp(Q)} + (1 + \theta_\tau \tau) \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} \frac{dp}{d\tau}$$

and when taxes are zero, we get:

$$\frac{dJ}{dt} = \frac{\theta_t}{\frac{\Lambda}{Q} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} (\rho_t - 1)$$

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau}{\frac{\Lambda}{Q} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} p(\rho_\tau - 1)$$

and so

$$MCPF_t = -\frac{\Lambda}{Q} \frac{\theta_t}{\frac{\Lambda}{Q} + q * mwtp(Q)} + (\rho_t - 1) \left(1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)}\right)$$
$$MCPF_\tau = -\frac{\Lambda}{Q} \frac{\theta_\tau}{\frac{\Lambda}{Q} + q * mwtp(Q)} + (\rho_\tau - 1) \left(1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)}\right)$$

Assuming $\theta_t = \theta_\tau$ and $\tau = t = 0$, note that $1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} = \left(\frac{q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)}\right)$. Therefore:

$$sign(MCPF_\tau - MCPF_t) = sign\left((\rho_\tau - \rho_t) * \frac{q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)}\right)$$
$$= sign\left(q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}}\right)$$

Finally, observe:

$$sign\left(\frac{1}{p}\frac{dJ}{d\tau}-\frac{dJ}{dt}\right)=sign\left((\rho_\tau-\rho_t)*\frac{1-\frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q}+q*mwtp(Q)}\right)$$
$$<0$$

□

Proof. ***Corollary 7.***

There are two cases. As a matter of terminology, we say the stability condition $\Delta = \left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(1+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{1}{\epsilon_{ms}}\right)>0$ does not restrict Λ if and only if $1+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{1}{\epsilon_{ms}}<0$.

Assume $\Delta>0$ does not restrict Λ. Then:

1. If there is overshifting of t (this is the case if $1-\frac{1}{\epsilon_{ms}}>0$), then $\rho_\tau>\rho_t$ implies $\frac{\Lambda_0\epsilon_D}{p_0q_0}>1$ and so $\frac{J}{q}\frac{\partial q}{\partial J}=-\omega_{SR}\left[1-\frac{\nu_q}{J}\left(1-\frac{1}{\epsilon_{ms}}\right)-\frac{\Lambda\epsilon_D}{(1+\tau)pq}\right]>0$.

2. If there is no overshifting of t (this is the case if $1-\frac{1}{\epsilon_{ms}}<0$), then $\epsilon_{ms}>0$ but $\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}<0$ implies $\epsilon_{ms}<0$ so we get a contradiction (this means assuming $\Delta>0$ does not restrict Λ implies overshifting of t).

3. Assume now that $\Delta>0$ does restrict Λ. Then $\rho_\tau>\rho_t$ implies $\frac{\Lambda_0\epsilon_D}{p_0q_0}>1$. Also $\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}>0$ implies

$$0<\Delta=\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\frac{\Lambda\epsilon_D}{(p(1+\tau)+t)q}\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)$$
$$<\frac{\nu_q}{J}\left[2-\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right]-\left(\frac{\nu_q}{J}+\frac{\epsilon_D^*-\frac{\nu_q}{J}}{\epsilon_S}+\frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)$$

Rearranging the expression we obtain $1-\frac{1}{\epsilon_{ms}}>0$ so that specific taxation is overshifted. Finally $\frac{J}{q}\frac{\partial q}{\partial J}=-\omega_{SR}\left[1-\frac{\nu_q}{J}\left(1-\frac{1}{\epsilon_{ms}}\right)-\frac{\Lambda\epsilon_D}{(1+\tau)pq}\right]>0$. □

C Connection to Pass-through Formulas in Delipalla and Keen (1992)

In this section, we show the connection of our results to Delipalla and Keen (1992). Note that in Delipalla and Keen (1992), the tax is on firms. The consumer price is defined as P, and the producer price is $P - t$.

In Delipalla and Keen (1992), "A" is defined as:

$$A \equiv -\frac{1}{\lambda}\frac{C_{xx}}{P_X}$$

where $\lambda \equiv \frac{dX}{dx_i}$.

Let usdefine $\epsilon_S = \frac{C_x}{xC_{ss}}$. Therefore, we can express "A" as

$$A = \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}$$

Next, Delipalla and Keen (1992) define "E" as:

$$E \equiv -\frac{P_{XX}X}{P_X}$$

Using the fact that $ms = -P_X s$, we can get $\frac{1}{\epsilon_{ms}} = 1 - E$. Thus,

$$E = 1 - \frac{1}{\epsilon_{ms}}$$

We then substitute for A and E using the expressions above in the pass-through expression in Delipalla and Keen (1992), and set $\tau = 0$

$$\rho_t = \frac{dP}{dt} = \frac{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s}}{2 - \frac{\lambda}{J} + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s} + \frac{\frac{\lambda}{J}}{\epsilon_{ms}}}$$

This is the same expression we have for a specific tax when consumers are fully optimizing and there are no pre-existing taxes (see Corollary 1).

Next, consider ad valorem taxes. Delipalla and Keen (1992) show the following:

$$\frac{dP}{d\tau} = \alpha \frac{dP}{dt}$$

where $\alpha \equiv \frac{P(1+A)+mc}{2+A}$. Thus, substituting in A yields:

$$\alpha = \frac{P(1 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}) + mc}{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}}$$

Therefore,

$$\begin{aligned}
\frac{dP}{d\tau} &= \frac{P(1 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}) + mc}{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}} \times \frac{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s}}{2 - \frac{\lambda}{J} + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s} + \frac{\frac{\lambda}{J}}{\epsilon_{ms}}} \\
&= \frac{P(1 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S}) + mc}{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s} - \frac{\lambda}{J}(1 - \frac{1}{\epsilon_{ms}})}
\end{aligned}$$

Therefore, the pass-through of ad valorem taxes is

$$\begin{aligned}
\rho_\tau &= \frac{1}{P}\frac{dP}{d\tau} \\
&= \frac{2 + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_S} - \frac{\frac{\lambda}{J}}{\epsilon_D}}{2 - \frac{\lambda}{J} + \frac{\epsilon_D^* - \frac{\lambda}{J}}{\frac{\lambda}{J}\epsilon_s} + \frac{\frac{\lambda}{J}}{\epsilon_{ms}}}
\end{aligned}$$

which is the expression we have for pass-through in Corollary 4.

D Microfoundations for Demand

In this section, we provide the microfoundation for parallel demands. First, we introduce a class of continuous choice models that are nested by our utility function.

Preferences. Let the representative consumer's utility function given by

$$u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m$$

for any $h_J : \{1, \ldots, J\} \to \mathbb{R}$ which is symmetric in all its arguments, continuously differentiable, strictly quasi-concave and $h(0, \ldots, 0) = 0$ and where the linear good m is interpreted as money.

Demand. The consumer's problem is

$$\max u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m \qquad (21)$$
$$\text{subject to } m + \sum_{j=1}^{J} p_j q_j = y.$$

When the consumer is facing symmetric prices $p_j = p$ for all j, we can transform the problem as follows. Define $H_J(Q) = h_J\left(\frac{Q}{J}, \ldots, \frac{Q}{J}\right)$ where we interpret Q as aggregate demand. The new problem then is given by

$$u^*(p, J, y) = \max_Q H_J(Q) + y - pQ.$$

From the first-order condition, we obtain the family of inverse demands $P(Q, J) = H'_J(Q)$. Furthermore, it is easy to see that given the optimal aggregate quantity $Q(p, J)$ for price p, the strict quasi-concavity of h_J implies the consumer chooses symmetric quantities $q_j = \frac{Q}{J}$ for all j in the original problem.

Furthermore, none of the assumptions on utility are too restrictive. We show that for any family of downward sloping aggregate demands there exists a utility function $u_J : \mathbb{R}^{J+1} \to \mathbb{R}$ satisfying the conditions above that rationalize the aggregate demands. Let $P(Q, J)$ be continuously differentiable and strictly decreasing in Q. Let H be any antiderivative $\int P(Q, J)dQ$, which exists because $P(Q, J)$ is differentiable. Then, for some $\rho \in (0, 1)$, the following is a strictly quasi-concave direct utility function that rationalizes $P(Q, J)$ for integer

J when all prices p_j in the market are equal:

$$u(q_1, \ldots, q_J, m) = H\left(\left(J^{\rho-1}\sum_{j=1}^{J} q_j^{\rho}\right)^{\frac{1}{\rho}}\right) + m.$$

Furthermore, we can make sense of J as a continuous variable if we permit a continuum of varieties $q : [0, J] \to \mathbb{R}$ and let

$$u_J(q, m) = H\left(\left(\int_0^J J^{\rho-1} q^{\rho}(j) dj\right)^{\frac{1}{\rho}}\right) + m.$$

We provide two examples in the following to further illustrate the idea of parallel demands and its applications.

Example 1. Bulow and Pfleiderer (1981) obtain the following three categories of inverse demands as the unique curves with the property of constant pass-through:

1. $P(Q, J) = \alpha_J - \beta_J Q^{\delta}$, for $\delta > 0$,
2. $P(Q, J) = \alpha_J - \beta_J log(Q)$,
3. $P(Q, J) = \alpha_J + \beta_J Q^{1/\eta}$, for $\eta < 0$, which is the constant elasticity inverse demand shifted by the intercept α_J.

An important case is when $\beta_J = \beta$ for all J, then the inverse aggregate demands are linearly separable in J and Q and they shift in parallel as J moves.[2] The fact that these are the only class of curves for which marginal costs are passed-on in a constant fraction makes them a tractable benchmark and therefore they have been popular in applied work. Fabinger and Weyl (2016) generalize Bulow and Pfleiderer (1983) and characterize a broader class of "tractable equilibrium forms" of the form $P(Q, J) = \alpha_J + \beta Q^t + \gamma Q^u$ which allow for greater

[2]For example, for the first class one possible family of utility functions, among many, that rationalize the inverse aggregate demands is given by

$$u_J(q_1, \ldots, q_J, m) = \alpha_J\left(J^{\rho-1}\sum_{i=1}^{J} q_i^{\rho}\right)^{\frac{1}{\rho}} - \beta_J\frac{\left(\sum_{i=1}^{J} q_i\right)^{\delta+1}}{\delta+1} + m.$$

modeling flexibility. Again, as long as β and γ are independent of J, then we say that the inverse demands shift in parallel.

Example 2. This example shows that our revealed-preference approach allows for rational preferences that display *hate-of-variety* ($a'(J) < 0$). Imagine there is a marginal cost of consumption cJ for each unit of some good that is consumed; that is, for each unit consumed, the agent faces a constant cost of evaluating each of J varieties before he chooses. Preferences are given by

$$U = H\left(\sum_{j=1}^{J} q_j\right) - cJ\sum_{j=1}^{J} q_j + m$$

where H is concave. The inverse demands are then $P(Q,J) = h(Q) - cJ$ with $h = H'$ decreasing, therefore aggregate demand shifts inward as the variety increases (the intercept being $h(0) - cJ$). We can interpret this as the agent displaying a strong degree of thinking aversion or attention costs. More generally, if the inverse demands are given by $P(Q,J) = a(J) - h(Q)$ then the sign of $a'(J)$ is unrestricted.

E Formulas in Calibration

Taking logs and rescaling by $\frac{W}{pQ}$ equation (15) we obtain the following expression which we use in Section 7 of the paper:

$$\frac{dlog(W)}{dlog(1+\tau)}\frac{W}{pQ} = \tilde{\Lambda}_0\frac{dlog(J)}{dlog(1+\tau)} - \frac{dlog(p)}{dlog(1+\tau)} + \theta_\tau\tau_0\frac{dlog(Q_L)}{dlog(1+\tau)} \tag{22}$$

where $\tilde{\Lambda}_0 \equiv \frac{\Lambda_0}{pQ}$.

We now show the derivation equation (23) in the paper. Note that the Lerner condition $\frac{p-mc}{p(1+\tau)} = \frac{\frac{\nu_q}{J}}{(1+\theta_\tau\tau)\epsilon_D}$ and the long-run free entry condition $\frac{\frac{dlogp}{d\tau}}{\frac{dlogq}{d\tau}} = -\frac{p-mc}{p}$ we can identify

$$\frac{\nu_q}{J} = -\epsilon_D\frac{1+\theta_\tau\tau}{1+\tau}\frac{\frac{dlogp}{d\tau}}{\frac{dlogq}{d\tau}} \tag{23}$$

We have from Proposition 2, and assuming constant mc, that

$$\frac{dJ}{d\tau} = -\frac{\theta_\tau J \epsilon_D}{(1+\tau)} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1 - \frac{\nu_q}{J}}{\epsilon_D^*}}{\triangle} \right]$$

and

$$\rho_\tau = \frac{\triangle - \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) + \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q} \left(\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right)}{\triangle}$$

where $\triangle \equiv 1 + \left[1 + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\frac{\nu_q}{J} \epsilon_S} \right] \left[1 - \frac{\Lambda \epsilon_D}{(1+\tau)pq} \right] - \frac{1}{\epsilon_{ms}} \frac{\Lambda \epsilon_D}{(1+\tau)pq} - \frac{\nu_q}{J} \left[1 - \frac{1}{\epsilon_{ms}} \right]$. Then

$$\triangle = -\frac{\theta_\tau J \epsilon_D}{(1+\tau)} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1 - \frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{dJ}{d\tau}} \right] = \frac{-\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) + \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q} \left(\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right)}{\rho_\tau - 1}$$

And so, using $\rho_\tau - 1 = (1+\tau)\frac{dlog(p)}{d\tau}$, then

$$\frac{\Lambda \epsilon_D}{pq} \left(\frac{1}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right) = -J \epsilon_D \frac{dlog(p)}{d\tau} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1 - \frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{dJ}{d\tau}} \right] + \frac{1+\tau}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right)$$

which implies

$$\frac{\Lambda}{pq} = -\frac{\epsilon_D^*}{\epsilon_D} (1 + \theta_\tau \tau) \frac{\frac{\epsilon_D}{\frac{\nu_q}{J}} \frac{dlog(p)}{d\tau}}{\frac{dlog(J)}{d\tau}} \left(1 + \frac{1}{\epsilon_{ms}} + \frac{1 - \frac{\nu_q}{J_0}}{\epsilon_D^*} \right) + (1+\tau) \frac{\epsilon_D^*}{\epsilon_D} \left(\frac{1}{\epsilon_D^*} - 1 + \frac{1}{\epsilon_{ms}} \right)$$

Now, from $\frac{\epsilon_D^*}{\epsilon_D} = \frac{1+\theta_\tau \tau}{1+\tau}$ and equation (23) we get

$$\frac{\Lambda}{pq} = -\frac{1+\theta_\tau \tau}{1+\tau}(1+\theta_\tau \tau)\frac{-\frac{1+\tau}{1+\theta_\tau\tau}\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+(1+\tau)\frac{1+\theta_\tau\tau}{1+\tau}\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right)$$
$$= (1+\theta_\tau\tau)\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+(1+\theta_\tau\tau)\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right)$$
$$= (1+\theta_\tau\tau)\left[\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}+\frac{1}{\epsilon_{ms}}-1\right]$$
$$= (1+\theta_\tau\tau)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}+1\right)+\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}-1\right]$$
$$= (1+\theta_\tau\tau)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\frac{dlog(Q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\right)+\frac{\frac{dlog(Q)}{d\tau}-\frac{dlog(J)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}-1\right]$$
$$= (1+\theta_\tau\tau)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\hat{\beta}^Q}{\hat{\beta}^J}\right)+\frac{\hat{\beta}^Q}{\hat{\beta}^J}\left(1+\frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}\right)+\frac{\frac{\nu_q}{J}}{\epsilon_D^*}-2\right]$$

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Online Appendix Table OA.1:
Effect of Food and Nonfood Sales Taxes **[Placebo Test]**

	(1)	(2)	(3)	(4)
Dependent variable : Prices (Panel A)				
Own tax rate differential	0.187		0.165	0.045
	(0.020)		(0.018)	(0.011)
Other tax rate differential		0.150	0.120	
		(0.021)	(0.018)	
Dependent variable : Quantity (Panel B)				
Own tax rate differential	-0.849		-0.853	-0.876
	(0.258)		(0.227)	(0.173)
Other tax rate differential		-0.132	0.022	
		(0.257)	(0.227)	
Dependent variable : Variety (Panel C)				
Own tax rate differential	-0.205		-0.215	-0.269
	(0.125)		(0.115)	(0.100)
Other tax rate differential		0.015	0.054	
		(0.106)	(0.093)	
Specification:				
Food dummy	y	y	y	y
Cell (border pair by year) fixed effects				y
N (observations)	8430	8430	8430	8430

Notes: This table reports regressions of prices, quantity and product variety on average tax rates for food and nonfood products. For each border pair-by-year cell there is two observations: one for food products and one for nonfood products. All variables are measured as within-cell differences average difference between the two contiguous counties. Own tax rate is the average food tax rate differential for food observations and the average nonfood tax rate differential for nonfood observations. Other tax rate is the average food tax rate differential for nonfood observations and the average nonfood tax rate differential for food observations. Standard errors are clustered at the border pair-by-year cell-level. Each regression includes a dummy variable for food products. Observations are weighted to reflect the number of underlying module-by-store-by-year observations in each cell.

Online Appendix Table OA.2: Sensitivity of parameter estimates to alternative values of demand elasticity and tax salience parameter

Panel A: Calibrated parameters					
Average tax rate, τ_0	0.034	0.034	0.034	0.034	0.034
Tax salience parameter, θ_τ	**0.528**	**0.475**	**0.581**	**0.528**	**0.528**
Demand elasticity, ϵ_D	**1.223**	**1.345**	**1.101**	**1.345**	**1.101**
Panel B: Reduced-form estimates					
Pass-through of taxes into pre-tax prices, $d\log(p)/d\log(1+\tau)$	0.039	0.039	0.039	0.039	0.039
Quantity response, $d\log(Q)/d\log(1+\tau)$	-0.731	-0.731	-0.731	-0.731	-0.731
Variety response, $d\log(J)/d\log(1+\tau)$	-0.243	-0.243	-0.243	-0.243	-0.243
Panel C: Model parameters estimated by matching reduced-form estimates					
Markup, $(p - c'(q))/p$	0.080	0.080	0.080	0.080	0.080
Implied conduct parameter, v_q/J	0.096	0.106	0.087	0.106	0.087
Inverse elasticity of marginal surplus, ϵ_{ms}	-0.936	-1.003	-0.877	-0.936	-0.936
Variety effect parameter, $\tilde{\Lambda}_0$	**0.133**	**0.127**	**0.191**	**-0.098**	**0.416**
Panel D: Calibrated welfare formulas					
Full marginal excess burden (MEB) formula, $d\tilde{W}/d\tau$	-0.085	-0.082	-0.100	-0.028	-0.153
Alternative MEB formula benchmarks:					
Harberger / Chetty-Looney-Kroft benchmark, $\theta_\tau * \tau_0 * d\log(Q)/d\log(1+\tau)$	-0.013				
Besley(1989)-style benchmark; i.e., full MEB formula with $\tilde{\Lambda}_0 = 0$	-0.052				

Notes: This table reports structural parameter estimates by finding parameters that allow the model to match the reduced-form estimates. The table reports sensitivity to different assumptions on the demand elasticity and the tax salience parameter. Columns (2) and (3) vary both parameters but hold the product of the tax salience parameter and demand elasticity constant. The last two columns only vary the demand elasticity.

Online Appendix Table OA.3: Additional Counterfactual Comparisons of Ad Valorem and Unit Tax Taxes

Variety effect parameter, $\tilde{\Lambda}_0$	Baseline variety effect estimate, $\tilde{\Lambda}_0 = 0.133$				No variety effect counterfactual, $\tilde{\Lambda}_0 = 0.000$		Large variety effect counterfactual, $\tilde{\Lambda}_0 = 1.000$		Very large variety effect counterfactual, $\Lambda_0 = 2.000$	
Inverse elasticity of marginal surplus, ϵ_{ms}	ϵ_{ms} = -0.936		ϵ_{ms} = -20.000		ϵ_{ms} = -20.000		ϵ_{ms} = -20.000		ϵ_{ms} = -20.000	
	Ad valorem tax ($d\tau$) (1)	Specific tax (dt) (2)	Ad valorem tax ($d\tau$) (3)	Specific tax (dt) (4)	Ad valorem tax ($d\tau$) (5)	Specific tax (dt) (6)	Ad valorem tax ($d\tau$) (7)	Specific tax (dt) (8)	Ad valorem tax ($d\tau$) (7)	Specific tax (dt) (8)
Panel A: Pass-through of taxes into pre-tax prices										
$d\log(p)/d\log(1+\tau)$ or $d\log(p)/dt$	0.039	0.058	0.010	0.030	0.006	0.028	0.080	0.069		
Difference between ad valorem and specific tax	-0.019		-0.020		-0.022		0.011			
Panel B: Marginal cost of public funds (*MCPF*)										
$MCFP_\tau$ or $MCPF_t$	0.083	0.067	0.107	0.088	0.017	0.040	1.611	0.880		
Difference between ad valorem and specific tax	0.017		0.019		-0.022		0.731			
Panel C: Additional Statistics										
$d\log(p)/d\log(1+\tau) \mid J$ or $d\log(p)/dt \mid J$	0.013	0.061	-0.040	0.003	-0.040	0.003	-0.040	0.003		
Difference between SR and LR pass-through	0.026	-0.003	0.050	0.028	0.046	0.026	0.120	0.066		
$d\log(J)/d\log(1+\tau)$ or $d\log(J)/dt$	-0.243	0.024	-0.628	-0.339	-0.578	-0.312	-1.416	-0.765		
$\partial\log(\pi)/\partial\log(1+\tau)$ or $\partial\log(\pi)/\partial t$	-0.041	0.004	-0.091	-0.048	-0.091	-0.048	-0.091	-0.048		
$\partial\log(p)/\partial\log(J)$	-0.108	-0.106	-0.084	-0.082	-0.083	-0.082	-0.088	-0.087		
$\partial\log(q)/\partial\log(J)$	-0.728	-0.717	-0.757	-0.745	-0.918	-0.903	0.290	0.285		
Stability condition (must be >0)	1.812	1.812	1.749	1.749	1.899	1.899	0.080	0.079	-0.348	-0.348

Notes: This table reports counterfactual estimates of reduced-form effects of specific taxes under different assumptions on variety effect and the inverse elasticity of marginal surplus, providing alternative scenarios reported in Table 5 using the model parameter estimates of Table 3. The final two columns do not report estimates since the large variety effect leads to a violation of stability condition. By contrast, the stability condition is satisfied for all of the columns in Table 5.

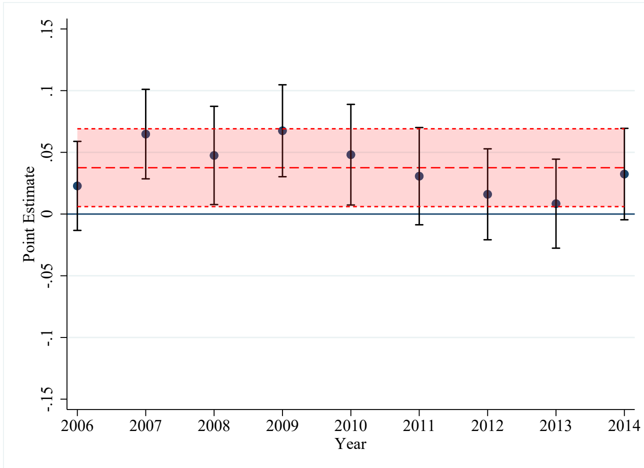
Online Appendix Table OA.4: Love-of-variety and long-run pass-through

Variety effect parameter, $\tilde{\Lambda}_0$	Baseline variety effect estimate, $\tilde{\Lambda}_0 = 0.133$				No variety effect counterfactual, $\tilde{\Lambda}_0 = 0.000$			
Inverse elasticity of marginal surplus, ϵ_{ms}	ϵ_{ms} = -0.936		ϵ_{ms} = -0.468		ϵ_{ms} = -0.936		ϵ_{ms} = -0.468	
	Ad valorem tax ($d\tau$) (1)	Specific tax (dt) (2)	Ad valorem tax ($d\tau$) (3)	Specific tax (dt) (4)	Ad valorem tax ($d\tau$) (5)	Specific tax (dt) (6)	Ad valorem tax ($d\tau$) (7)	Specific tax (dt) (8)
Panel A: Pass-through of taxes into pre-tax prices								
$d\log(p)/d\log(1+\tau)$ or $d\log(p)/dt$	0.0390	0.0580	0.0670	0.0849	0.0355	0.0584	0.0701	0.0939
Difference between ad valorem and specific tax	-0.019		-0.018		-0.023		-0.024	
Panel B: Marginal cost of public funds (*MCPF*)								
$MCFP_\tau$ or $MCPF_t$	0.083	0.067	0.061	0.046	0.047	0.070	0.082	0.106
Difference between ad valorem and specific tax	0.017		0.015		-0.023		-0.024	
Panel C: Additional Statistics								
$d\log(p)/d\tau \mid J$ or $d\log(p)/dt \mid J$	0.013	0.061	0.084	0.137	0.013	0.061	0.084	0.137
Difference between SR and LR pass-through	0.026	-0.003	-0.017	-0.052	0.022	-0.002	-0.013	-0.043
$d\log(J)/d\log(1+\tau)$ or $d\log(J)/dt$	-0.243	0.024	0.133	0.378	-0.244	0.024	0.142	0.418
$\partial\log(\pi)/\partial\log(1+\tau)$ or $\partial\log(\pi)/\partial t$	-0.041	0.004	0.024	0.073	-0.041	0.004	0.024	0.073
$\partial\log(p)/\partial\log(J)$	-0.108	-0.106	-0.139	-0.137	-0.092	-0.091	-0.104	-0.102
$\partial\log(q)/\partial\log(J)$	-0.728	-0.717	-0.691	-0.680	-0.907	-0.893	-0.893	-0.879
Stability condition (must be >0)	1.812	1.812	1.877	1.877	1.801	1.801	1.698	1.698

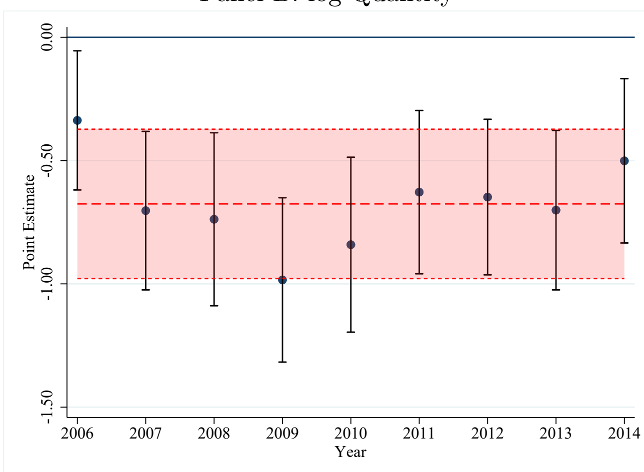
Notes: This table reports counterfactual estimates of redued-form effects of unit taxes under different assumptions on the variety effect and the inverse elasticity of marginal surplus, providing alternative scenarios reported in Table 5 using the model parameter estimates of Table 3.

Figure OA.1: Year-by-Year OLS Regression Coefficients

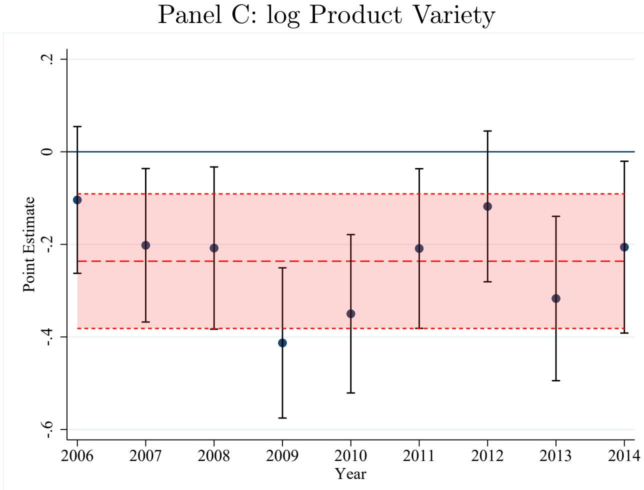
Panel A: log Prices



Panel B: log Quantity

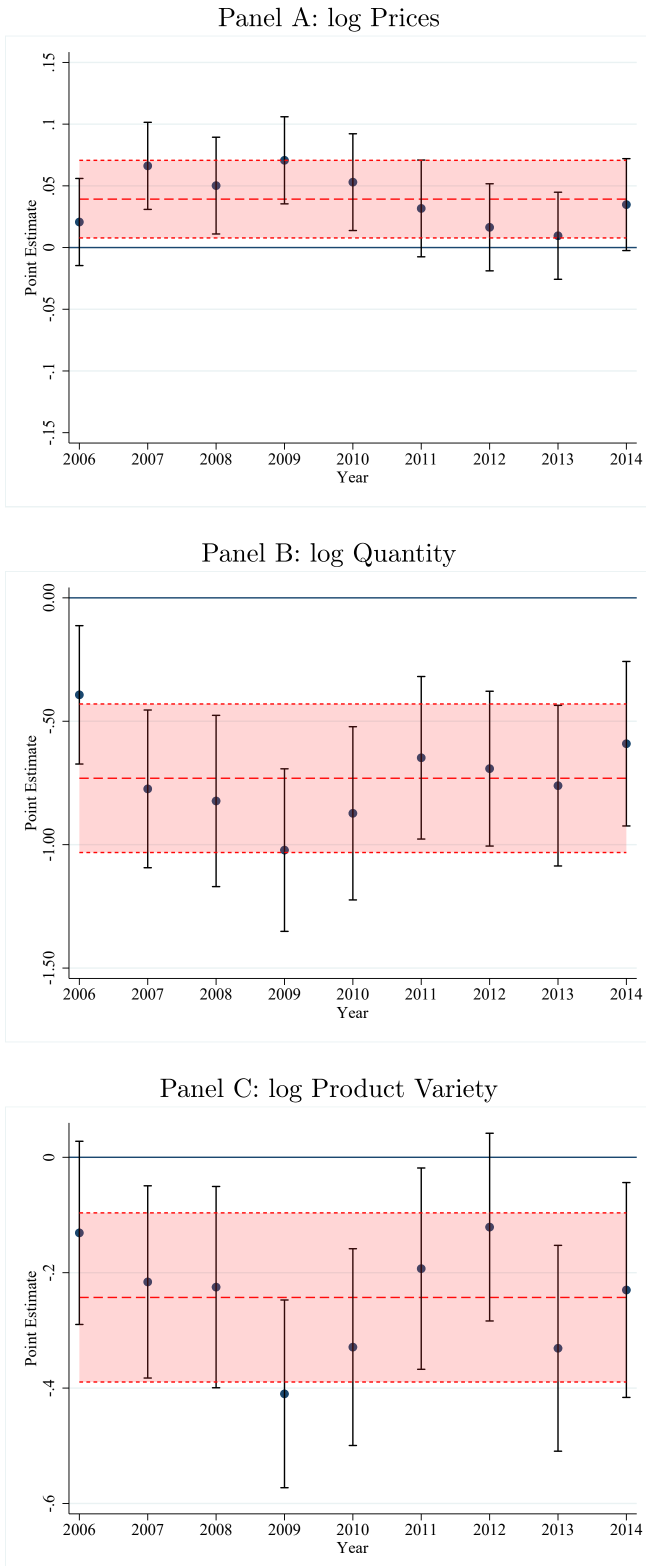


Panel C: log Product Variety



Notes: This figures shows yearly estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product varity (C). All models are based on equation (22) and estimated by OLS. The black vertical bars indicate 95% confidence intervals. The dashed red horizontal line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.

Figure OA.2: Year-by-Year 2SLS Regression Coefficients



Notes: This figures shows yearly estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product varity (C). All models are based on equation (22) and estimated by 2SLS. The instrument is the average state-level, leave-county-out average tax rate for each module-year cell. The black vertical bars indicate 95% confidence intervals. The dashed red horizontal line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.